Numerical investigation of the transient heat transfer in the inviscid stagnation flow with solidification

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Abstract *The problem of transient heat transfer and growth of solid in the inviscid stagnation flow when phase change from liquid to solid occurs is considered. A fast and accurate numerical scheme is developed to determine the instantaneous temperature distribution in both solid and liquid phases and the growth rate of solid directly, without iterative calculation. The solution of the dimensionless governing equations is dependent on the three dimensionless parameters. The characteristics of the transient heat transfer and solidification process for all the parameters are elucidated.*

Nomenclature

A = potential flow strain rate
C_S = specific heat of solid
$erf(x)$ = error function
$erfc(x)$ = complementary error function, $1 - erf(x)$
K = thermal conductivity
K_R = ratio of thermal conductivity, K_S/K_L
L = latent heat
$\text{Log}(x)$ = $\text{Log}_{10}(x)$
$Q_S(t)$ = heat flux at the surface of solid, $K_S[\partial T_S(y,t)/\partial y]$ at $y = 0$
$Q_L(t)$ = heat flux at the liquid side of solid-liquid interface, $K_L[\partial T_L(y,t)/\partial y]$ at $y = Y(t)$
Q_{steady} = steady-state heat flux, $Q_S(t) = Q_L(t) = Q_{steady}$ at $t \to \infty$
Ste = Stefan number, $C_S(T_F - T_C)/L$
T = temperature
T_C = cold temperature of solid surface
T_F = freezing temperature of liquid
T_H = hot temperature of liquid
t = time
υ = vertical velocity component of fluid
$Y(t)$ = thickness of solid phase
Y_{eq} = solid thickness at equilibrium state
y = axial coordinate

Greek symbols

α = thermal diffusivity
α_R = ratio of thermal diffusivity, α_S/α_L
$\delta(\tau)$ = dimensionless solid thickness
δ_{eq} = dimensionless solid thickness at equilibrium state
ζ = dimensionless axial coordinate, $\sqrt{A/\alpha_L}\, y$
η = transformed coordinate, $\zeta/\delta(\tau)$
θ_L = dimensionless temperature in liquid region, $(T_L - T_H)/(T_F - T_H)$
θ_S = dimensionless temperature in solid region, $(T_S - T_C)/(T_F - T_C)$
θ_R = ratio temperature, $(T_H - T_F)/(T_F - T_C)$
ρ = density
σ = growth parameter in Neumann problem
τ = dimensionless time, At

Subscripts

L = liquid
S = solid
∞ = infinity

1. Introduction

Heat transfer problems in the freezing process have attracted considerable attention in view of both their theoretical interest and their practical applications such as crystal growth, casting, welding, and spray forming. Since the work by Stefan (Carslaw and Jaeger, 1959), many authors have studied the phase-change problems without fluid flow (Muehlbauer and Sunderland, 1965; Hsieh, 1995), and with forced convection (Savino and Siegel, 1969; Epstein, 1976) or natural convection (Benard *et al.*, 1986) in the melt. In the problems with forced convection, most of the studies assumed the rate of convective heat transfer from the liquid side of solid-liquid interface as a constant, and determined the temperature distribution in solid and the location of solid-liquid interface (Savino and Siegel, 1969; Epstein, 1976). However, the phase-change process can be affected by the transient development of temperature distribution in liquid and vice versa. Recently, Yoo (1991, 1997) considered the transient behaviour of the temperature distribution in both solid and liquid phases and the freezing rate in the rotating-disk-revolving-fluid system. On the other hand, Rangel and Bian (1995, 1996) and Yoo (1999) investigated an inviscid stagnation-flow solidification problem. Both the rotating-disk-revolving-fluid system and the stagnation-flow solidification problem have forced fluid flows towards the solid-liquid interface, and accordingly many similar characteristics were found.

In this study, we consider the inviscid stagnation-flow solidification problem (Figure 1), which is important in engineering applications such as solidification of a droplet impinging on a cold substrate in spray processes. The fluid flows toward the solid. Initially, the fluid is kept at a uniform temperature, and the temperature of the substrate is suddenly lowered to the temperature below the freezing point and maintained constant. As a result, solidification occurs at the surface of the solid, and the solid grows with time. If there is no fluid flow, then this problem becomes the well-known Stefan problem with Neumann's solution (Carslaw and Jaeger, 1959). Rangel and Bian studied this problem with an iterative numerical method (Rangel and Bian, 1995), and with a method of quasi-steady approximation (Rangel and Bian, 1996). They focused the main attention on the growth of solid and the existence of an asymptotic limit of the solid thickness. On the other hand, Yoo (1999) obtained an analytic solution at
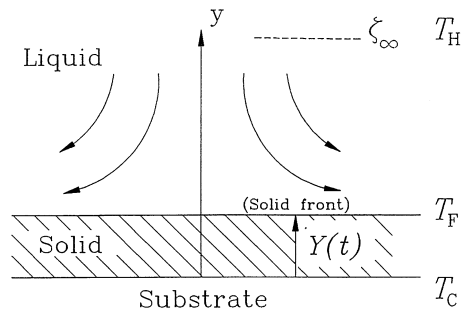
Figure 1.
Definition sketch

the initial stage of freezing by expanding it in powers of time, and showed the effect of stagnation flow on the pure conduction problem of Neumann (Carslaw and Jaeger, 1959).

The main physical quantities in the present problem are the growth of solid and the heat transfer rate at the surface of solid and the liquid side of solid-liquid interface. At first, we obtain the governing equations expressed with the three dimensionless parameters of θ_R/K_R, Ste, and α_R. And we develop a fast and accurate numerical scheme using a body-fitted coordinate. The numerical scheme directly determines the instantaneous temperature distribution and the solid thickness, without iterative calculation, and allows large time steps. The characteristics of the heat transfer and growth of solid for all the variables are elucidated.

2. Governing equations and numerical method

We consider the inviscid stagnation flow impinging on the surface of solid (Figure 1). Initially $(t = 0)$, the fluid is kept at a uniform temperature $T_H(T_H > T_F)$. For $t > 0$, the temperature of the substrate is suddenly lowered to $T_C(T_C < T_F)$ and maintained constant. We assume that the thermophysical properties of solid and liquid phases are constant, and the density change of the material upon freezing is neglected so that there is no fluid flow induced by the volumetric change in the phase-change process. Under these assumptions, the energy equations (Yoo, 1991; Rangel and Bian, 1996) governing the temperature distributions in the solid and liquid phases can be written as

$$\frac{\partial T_S}{\partial t} = \alpha_S \frac{\partial^2 T_S}{\partial y^2} \text{ at } 0 < y < Y(t) \tag{1}$$

$$\frac{\partial T_L}{\partial t} - 2A[y - Y(t)]\frac{\partial T_L}{\partial y} = \alpha_L \frac{\partial^2 T_L}{\partial y^2} \text{ at } y > Y(t) \tag{2}$$

In equation (2), the second term represents the convective term created by the inviscid stagnation flow with vertical velocity component of $v = -2A[y - Y(t)]$. At the solid-liquid interface, $y = Y(t)$, where the change of state occurs, the energy balance (Carslaw and Jaeger, 1959) is maintained:

$$K_S \frac{\partial T_S}{\partial y} - K_L \frac{\partial T_L}{\partial y} = \rho L \frac{dY}{dt} \text{ at } y = Y(t) \tag{3}$$

Additional boundary conditions are

$$T_S = T_C \text{ at } y = 0, T_S = T_L = T_F \text{ at } y = Y(t), T_L = T_H \text{ at } y \to \infty \tag{4}$$

In addition, prior to the onset of solidification, the temperature of liquid is uniform and is equal to T_H. The heat fluxes at the surface of solid (Q_S) and the liquid side of solid-liquid interface (Q_L) are defined as

$$Q_S(t) = -K_S \left[\frac{\partial T_S(y,t)}{\partial y} \right] \text{ at } y = 0 \tag{5}$$

$$Q_L(t) = -K_L \left[\frac{\partial T_L(y,t)}{\partial y} \right] \text{ at } y = Y(t) \tag{6}$$

$Q_S(t)$ and $Q_L(t)$ are of the same value, $Q_S(t) = Q_L(t) = Q_{steady}$, at $t \to \infty$.
Introducing the following dimensionless variables

$$\tau = At, \zeta = \sqrt{\frac{A}{\alpha_L}} y, \theta_S = \frac{T_S - T_C}{T_F - T_C}, \theta_L = \frac{T_L - T_H}{T_F - T_H}$$

$$K_R = K_S/K_L, \quad \alpha_R = \alpha_S/\alpha_L, \quad \theta_R = \frac{T_H - T_F}{T_F - T_C}, \quad Ste = \frac{C_S(T_F - T_C)}{L} \tag{7}$$

the governing equations (1)-(4) are written as

$$\frac{\partial \theta_S}{\partial \tau} = \alpha_R \frac{\partial^2 \theta_S}{\partial \zeta^2} \text{ at } 0 < \zeta < \delta \tag{8}$$

$$\frac{\partial \theta_L}{\partial \tau} - 2(\zeta - \delta) \frac{\partial \theta_L}{\partial \zeta} = \frac{\partial^2 \theta_L}{\partial \zeta^2} \text{ at } \zeta > \delta \tag{9}$$

$$\frac{\partial \theta_S}{\partial \zeta} + \frac{\theta_R}{K_R} \frac{\partial \theta_L}{\partial \zeta} = \frac{1}{\alpha_R Ste} \frac{d\delta}{d\tau} \text{ at } \zeta = \delta \tag{10}$$

$$\theta_S = 0 \text{ at } \zeta = 0, \quad \theta_S = \theta_L = 1 \text{ at } \zeta = \delta(\tau), \quad \theta_L = 0 \text{ at } \zeta \to \infty \tag{11}$$

where

$$\delta(\tau) = \sqrt{\frac{A}{\alpha_L}} Y(t) \tag{12}$$

denotes the dimensionless solid thickness. It is assumed that $\delta(0) = 0$.

It is to be noted that θ_R/K_R is one parameter in the dimensionless governing equations (8)-(11). It is because the heat transfer rate is determined by Fourier's law of heat conduction. The parameter of $\theta_R/K_R = K_L(T_H - T_F)/K_S(T_F - T_C)$ represents the ratio of the heating effect of the liquid to the cooling effect to freeze the liquid by the cold substrate. There are several numerical methods for the moving boundary problems (Crank, 1981). In this study, the unsteady solution is found by applying the finite difference method (Crank, 1981; Sparrow et al., 1978) after fixing the moving boundary for all times by a coordinate transformation. Let us introduce the following coordinate

transformation to fix the moving boundary of the position of the solid-liquid interface $\zeta = \delta(\tau)$ at $\eta = 1$.

$$\eta = \frac{\zeta}{\delta(\tau)} \tag{13}$$

Equations (8)-(11) are transformed as

$$\delta^2 \frac{\partial \theta_S}{\partial \tau} - \frac{\eta}{2} \frac{d\delta^2}{d\tau} \frac{\partial \theta_S}{\partial \eta} = \alpha_R \frac{\partial^2 \theta_S}{\partial \eta^2} \text{ at } 0 < \eta < 1 \tag{14}$$

$$\delta^2 \frac{\partial \theta_L}{\partial \tau} - \frac{\eta}{2} \frac{d\delta^2}{d\tau} \frac{\partial \theta_L}{\partial \eta} - 2(\eta - 1)\delta^2 \frac{\partial \theta_L}{\partial \eta} = \frac{\partial^2 \theta_L}{\partial \eta^2} \text{ at } \eta > 1 \tag{15}$$

$$\frac{\partial \theta_S}{\partial \eta} + \frac{\theta_R}{K_R} \frac{\theta_L}{\partial \eta} = \frac{1}{2\alpha_R Ste} \frac{d\delta^2}{d\tau} \text{ at } \eta = 1 \tag{16}$$

$$\theta_S = 0 \text{ at } \eta = 0, \quad \theta_S = \theta_L = 1 \text{ at } \eta = 1, \quad \theta_L = 0 \text{ at } \eta \rightarrow \infty \tag{17}$$

The instantaneous heat fluxes at the surface of solid and the liquid side of solid-liquid interface are expressed as the following equations:

$$\frac{Q_S(\tau)}{Q_{steady}} = \frac{\sqrt{\pi} K_R}{2\delta \theta_R} \frac{\partial \theta_S}{\partial \eta} \text{ at } \eta = 0 \tag{18}$$

$$\frac{Q_L(\tau)}{Q_{steady}} = -\frac{\sqrt{\pi}}{2\delta} \frac{\partial \theta_L}{\partial \eta} \text{ at } \eta = 1 \tag{19}$$

It can be shown (Yoo, 1997, 1999) that the solution in the limit of $\tau \rightarrow 0$ is the Neumann's solution (Carslaw and Jaeger, 1959) by letting $\theta_{S,L} = \theta_{S,L}[\zeta(2\sqrt{\tau})]$:

$$\theta_S(\eta) = \frac{erf(\sigma \eta)}{erf(\sigma)} \tag{20}$$

$$\theta_L(\eta) = \frac{erfc(\sigma \sqrt{\alpha_R} \eta)}{erfc(\sigma \sqrt{\alpha_R})} \tag{21}$$

$$\delta(\tau) = 2\sigma \sqrt{\alpha_R \tau} \tag{22}$$

$$\frac{\exp(-\sigma^2)}{erf(\sigma)} - \frac{\theta_R \sqrt{\alpha_R} \exp(-\sigma^2 \alpha_R)}{K_R erfc(\sigma \sqrt{\alpha_R})} = \sqrt{\frac{\pi \sigma}{Ste}} \tag{23}$$

The solution of equations (20)-(23) at a sufficiently small time of $\tau_0 = 10^{-5}$ is used as the initial field to start the numerical calculation. An implicit finite-difference scheme was used for the energy equations (14) and (15). To solve the resulting difference equations for the temperature distributions at τ^{j+1}, the values of δ^2 and $d\delta^2/d\tau = 2\delta(d\delta/d\tau)$ at τ^{j+1} are needed as input. The value of $(d\delta/d\tau)^{j+1}$ is obtained from equation (16) by introducing $\partial\theta_S/\partial\eta$, ∂_L/∂ and δ which are known at τ^j. And if $\partial\theta_S/\partial\eta$ and $\partial\theta_L/\partial\eta$ in equation (16) are regarded as constants during the interval τ^j to τ^{j+1} and equal to their values at τ^j, then this equation can be written as

$$\delta \ d\delta = G^j d\tau, \quad G^j : \text{constant at } \tau^j$$

which gives

$$\delta^{j+1} = \sqrt{(\delta^j)^2 + 2G^j(\tau^{j+1} - \tau^j)} \tag{24}$$

The calculated $\delta^j + 1$ in equation (24) is used as an input value to the finite difference equations.

One hundred grid points were developed uniformly throughout the solid region, and 500 grid points were developed non-uniformly throughout the liquid region according to the relation $\eta_i = \eta_\infty[(i - 1)/(N - 1)]^{1.5}$, where N is the total number of grid points in the liquid region. The resulting difference equations for the temperature distribution were solved non-iteratively at each time step by using the tridiagonal matrix algorithm. The accuracy of the numerical scheme was checked with the exact solution of Neumann (Carslaw and Jaeger, 1959) and the exact steady-state solution of equations (8)-(11), and the scheme was proved to be fast and accurate. In solving the Neumann problem, agreement with the exact solution to within 0.2 percent was attained for the solid thickness $Y(t)$. When there was fluid flow, the solution approached the exact steady-state solution as time went on. At the initial stage of solidification, the very small time step of $\Delta\tau = 10^{-6}/Ste$ was used, and the time step was sequentially increased to a certain maximum value depending on the parameters. The implicit finite-difference scheme applied to the energy equations (14) and (15) allows large time steps. Thus, the numerical scheme is very fast, since the temperature distribution and the solid thickness at each time step are determined directly without iterative calculation. The real computation time to calculate up to $\delta(\tau) = 0.99\delta_{eq}$ was approximately $1 \sim 3$ minutes, with pentium personal computer (90MHz).

3. Results and discussion
The governing equations (8)-(11) show that the solution of the present problem is dependent on the three dimensionless parameters of θ_R/K_R, Ste, and α_R. And we make the parametric study for the characteristics of the growth of solid and heat transfer in the range of $0.1 \leq \theta_R/K_R \leq 10$, $0.01 \leq Ste \leq 1$, and

$0.1 \leq \alpha_R \leq 10$. In most of the phase change problems such as crystal growth, casting, and welding, the Stefan number is less than 1, since the latent heat is very large in general.

At first, the thickness of solid, $\delta(\tau)$, for several values of θ_R/K_R, Ste, and α_R are presented in Figure 2. The solid thickness increases monotonously with time, and approaches a final equilibrium state. The growth rate of solid is increased as α_R or Ste increases, but is decreased as θ_R/K_R increases. Figure 2
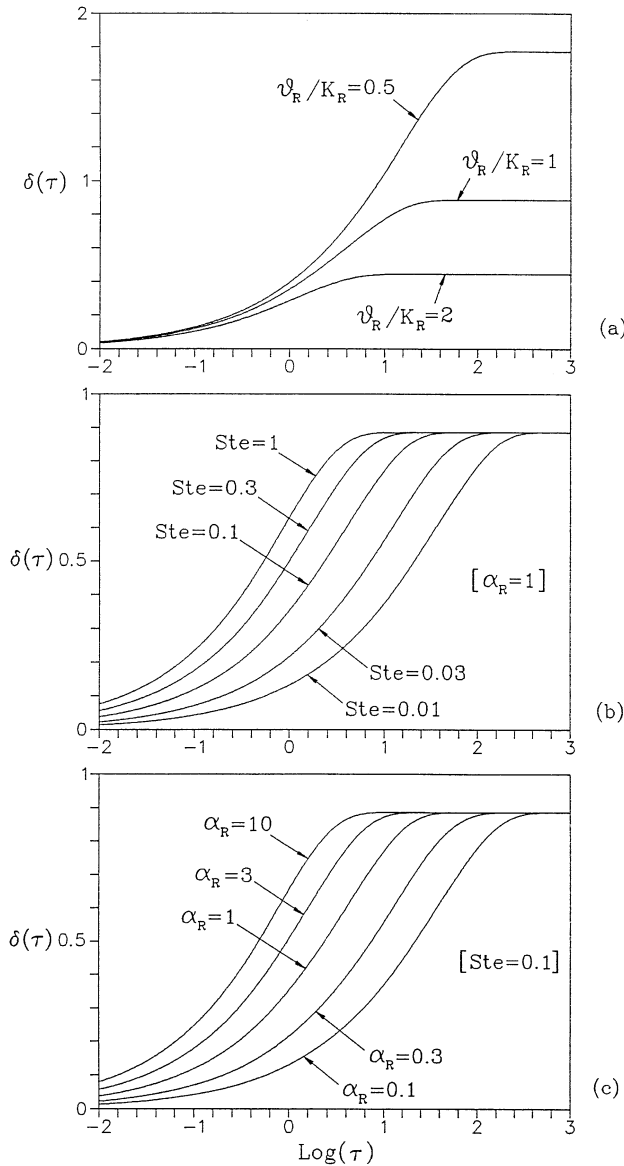
Figure 2.
Dimensionless solid
thickness with time:
(a) $\theta_R/K_R = 0.5$, 1 and 2
with $Ste = 0.1$ and
$\alpha_R = 1$; (b) $Ste = 0.01$,
0.03, 0.1, 0.3 and 1 with
$\theta_R/K_R = 1$ and $\alpha_R = 1$;
(c) $\alpha_R = 0.1$, 0.3, 1, 3 and
10 with $\theta_R/K_R = 1$ and
$Ste = 0.1$

shows the existence of an asymptotic limit of the solid thickness. In the Neumann problem (Carslaw and Jaeger, 1959) without fluid flow, there is no mechanism to stop the propagation of a temperature wave emanating from the solid-liquid interface, and the solid grows continuously with time according to the relation $Y(t) = 2\sqrt{\alpha_S t}$. When there is forced fluid flow towards the solid-liquid interface, however, the fluid flow towards the solid restricts the propagation of the wave in the liquid, and consequently the system approaches a final equilibrium state as time goes on. The equilibrium state can be obtained from the steady-state governing equations. Equations (8)-(11) with $\partial/\partial\tau = 0$ yield the following equilibrium state:

$$\theta_S(\zeta) = \frac{\zeta}{\delta_{eq}}, \quad \theta_L(\zeta) = erfc(\zeta - \delta_{eq}), \quad \delta_{eq} = \sqrt{\frac{\pi}{2}\frac{K_R}{\theta_R}} \qquad (25)$$

The asymptotic limit of the solidification front (δ_{eq}) shown in Figure 2 and equation (25) shows that the equilibrium solid thickness is dependent on the one parameter of θ_R/K_R, but is independent of Ste and α_R. The Stefan number and the thermal diffusivity ratio are only time-governing parameters of the solidifying process, but do not affect the final equilibrium state. On the other hand, Rangel and Bian (1995, 1996) concluded that the temperature distribution and the solid thickness for a large value of time ($\tau \rightarrow \infty$) are independent of Ste, while changes in the parameters θ_R, α_R and K_R affect the long-time behaviour of the solution. For the parameters of Ste and α_R, Figure 2(b),(c) shows that $\delta(\tau)'$s for $Ste = 0.01, 0.03, 0.1, 0.3$, and 1 with $\alpha_R = 1$ are nearly identical to those for $\alpha_R = 0.1, 0.3, 1, 3$, and 10 with $Ste = 0.1$, respectively. At the initial stage of solidification ($\tau \ll 1$), the conduction solution of equation (22) is valid, and we can express the solid thickness as $\delta(\tau) = F(\alpha_R, Ste)\sqrt{\tau}$, for a given value of θ_R/K_R. For the parameters in Figures 2(b),(c), the relative error between $F_1 = F(1, 0.01)$ and $F_2 = F(0.1, 0.1)$ at which $\alpha_R Ste = 0.01$ are $|F_1 - F_2|/F_1 = 0.027$, and that between $F_3 = F(1, 1)$ and $F_4 = F(0.1, 10)$ at which $\alpha_R Ste = 1$ are $|F_3 - F_4|/F_3 = 0.12$. As time goes on, the difference in the solid thickness $\delta(\tau)$s for different values of α_R and Ste with $\alpha_R Ste = constant$ decreases, since $\delta(\tau)$s have the same equilibrium value, if $\theta_R/K_R = constant$ (Figures 2(b),(c)). We can see that $\delta(\tau)$ is approximately a function of the two parameters, θ_R/K_R and $\alpha_R Ste$.

Second, the transient heat fluxes at the surface of solid ($Q_S(\tau)$) and the liquid side of solid-liquid interface ($Q_L(\tau)$) are presented in Figures 3-5. Figure 3 shows the heat fluxes for $\theta_R/K_R = 0.1, 1$ and 10. As θ_R/K_R increases, both $Q_S(\tau)/Q_{steady}$ and $Q_L(\tau)/Q_{steady}$ are decreased, that is, the response time of heat transfer in both solid and liquid phases is decreased, since the maximum solid thickness (δ_{eq}) that can be grown is inversely proportional to θ_R/K_R (equation (25)).

The dependency of the heat fluxes on Ste is shown in Figure 4. Figure 4(a) represents $Q_L(\tau)/Q_{steady}$ for $Ste = 0.01, 0.03, 0.1, 0.3, 1$, and the case with no phase change. The transient stagnation point heat transfer for viscous fluid

without phase change was investigated in the early years (Chao and Jeng, 1965; Sano, 1981). In Figure 4(a), the curve of no phase change was obtained by solving the equations for the transient heat transfer in inviscid stagnation flow. When phase change is present, $Q_L(\tau)/Q_{steady}$ is larger than that of the case with no phase change. And the difference is increased, as *Ste* increases. The forced fluid flow towards the solid-liquid interface inhibits the propagation of a temperature front in the liquid region. If the solid grows more rapidly for larger *Ste*, the propagation of the temperature front is more strongly inhibited. And consequently, it tends to reduce the speed of approach to equilibrium state. On the other hand, Figure 4(b) shows that $Q_S(\tau)/Q_{steady}$ is decreased as *Ste*
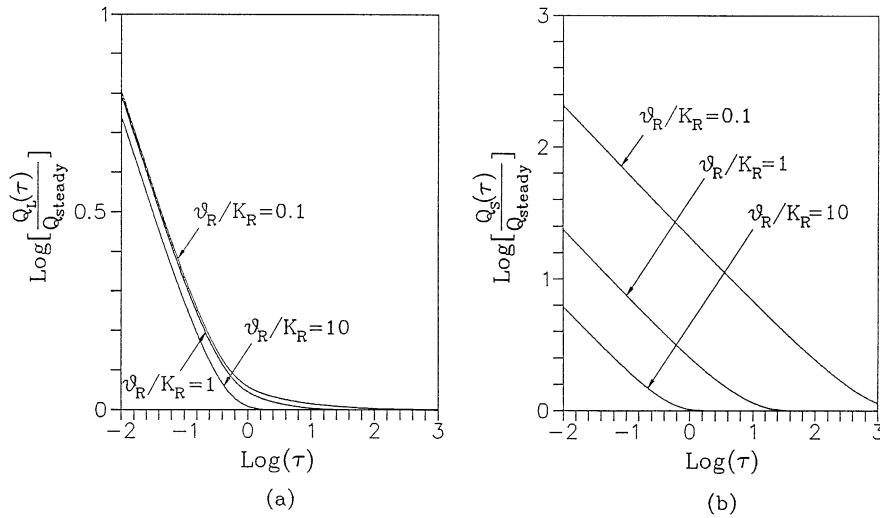
Figure 3.
Transient heat fluxes at the surface of solid, $Q_S(\tau)$, and the liquid side of solid-liquid interface, $Q_L(\tau)$, for several values of θ_R/K_R with *Ste* = 0.1 and $\alpha_R = 1$:
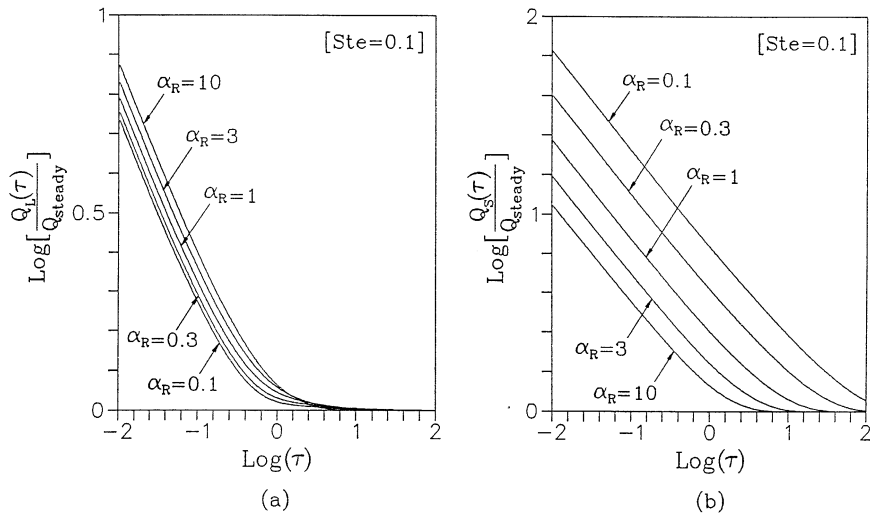(a) $Q_L(\tau)/Q_{steady}$;
(b) $Q_S(\tau)/Q_{steady}$
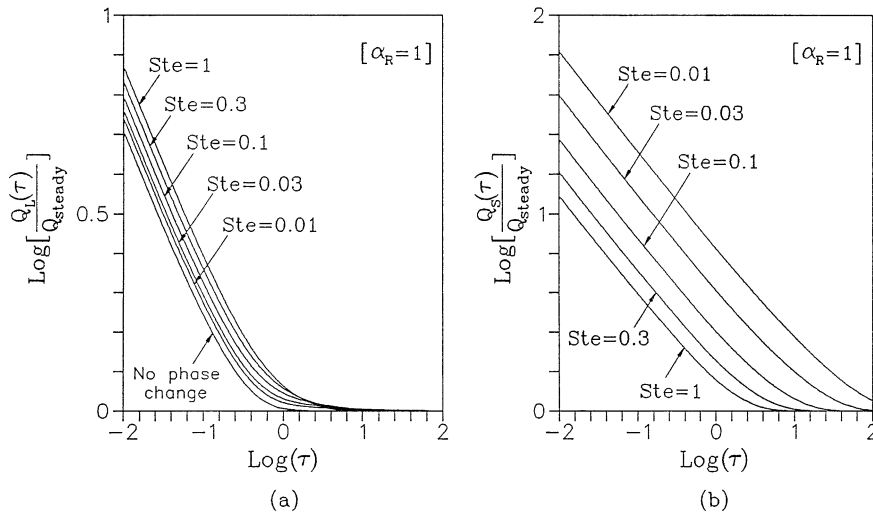
Figure 4.
Transient heat fluxes at the surface of solid, $Q_S(\tau)$, and the liquid side of solid-liquid interface, $Q_L(\tau)$, for several Stefan numbers with $\alpha_R = 1$ and $\theta_R/K_R = 1$ and $Q_L(\tau)$ for the case with no phase change:
(a) $Q_L(\tau)/Q_{steady}$;
(b) $Q_S(\tau)/Q_{steady}$

increases. It is because the equilibrium solid thickness is independent of *Ste*, and the solid thickness ($\delta(\tau)$) reaches its quasi-steady state more rapidly for larger *Ste* (Figure 2).

Figure 5 shows the heat fluxes for several values of α_R. The equilibrium state is independent of α_R, and the solid grows faster for larger α_R, as is the case with *Ste* (Figure 2). And accordingly, the dependency of heat fluxes on the variable α_R shows the same characteristics as that for *Ste*. Comparing Figure 5 with Figure 4, it can be also observed that the heat fluxes (Figure 4) for $Ste = 0.01, 0.03, 0.1, 0.3$, and 1 with $\alpha_R = 1$ are nearly identical to those (Figure 5) for $\alpha_R = 0.1, 0.3, 1, 3$, and 10 with $Ste = 0.1$, respectively. That is, the heat fluxes $Q_S(\tau)$ and $Q_L(\tau)$ are nearly unvarying for the changes in α_R and *Ste*, if $\alpha_R Ste = constant$, as with solid thickness $\delta(\tau)$ in Figure 2.

In the above, the characteristics of the instantaneous solid thickness and the transient heat transfer were investigated. We have observed that the combined quantity of $Y(\tau)$ and $Q_S(\tau)$, $Q_S(\tau)Y(\tau)$, has a notable characteristic during the phase-change process. In Figure 6, the behaviour of the function, $Log[Q_S(\tau)Y(\tau)/Q_{steady}Y_{eq} - 1]$, is presented for several values of θ_R/K_R and *Ste*. In Figure 6(a), the range of the value of $F(\tau) = [Q_S(\tau)/Q_{steady}][Y(\tau)/Y_{eq}]$ for $\theta_R/K_R = 0.1$ is $1 < F(\tau) < 1.016$ and that for $\theta_R/K_R = 10$ is $1 < F(\tau) < 1.0018$. And in Figure 6(b), the ranges for $Ste = 0.01$ and $Ste = 1$ are $1 < F(\tau) < 1.0016$ and $1 < F(\tau) < 1.048$, respectively. It can be seen that $Q_S(\tau)/Q_{steady} \approx Y_{eq}/Y(\tau)$ throughout the freezing process. The heat transfer rate at the surface of solid can be investigated experimentally by measuring the sublimation rate from naphthalene-coated surface and using the analogy between heat and mass transfer (Tien and Campbell, 1963). The result for $[Q_S(\tau)/Q_{steady}][Y(\tau)/Y_{eq}]$ in Figure 6 implies that the transient heat flux at the
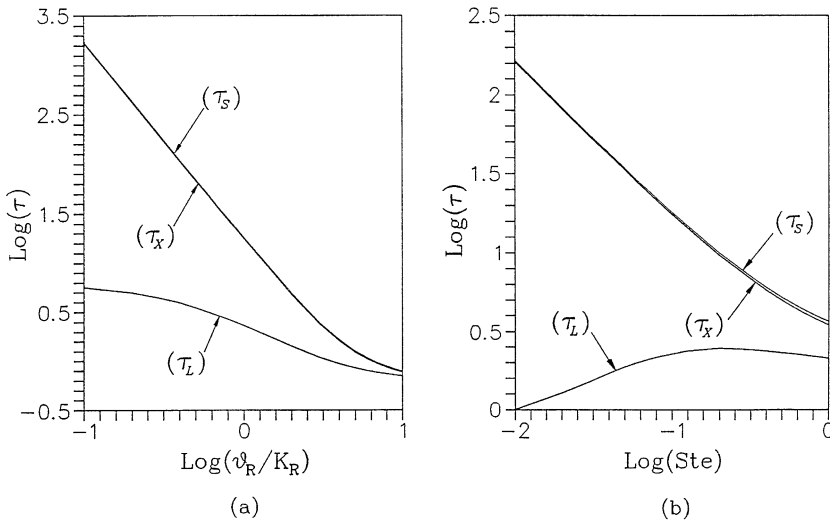

Figure 5.
Transient heat fluxes at the surface of solid, $Q_S(\tau)$, and the liquid side of solid-liquid interface, $Q_L(\tau)$, for several values of α_R with $\theta_R/K_R = 1$ and $Ste = 0.1$:
(a) $Q_L(\tau)/Q_{steady}$;
(b) $Q_S(\tau)/Q_{steady}$

surface of solid ($Q_S(\tau)$) can be obtained with sufficient accuracy by measuring the instantaneous thickness of solid ($Y(\tau)$) or vice versa, especially for large values of θ_R/K_R or small Ste.

As a final observation on the effect of the parameters, their influence on the time required to reach the quasi-steady state is shown in Figure 7 as functions of θ_R/K_R and Ste. In the Figure, curves (τ_X), (τ_S), and (τ_L) represent the time at which $Y(\tau_X) = 0.95Y_{eq}$, $Q_S(\tau_S) = 1.05Q_{steady}$, and $Q_L(\tau_L) = 1.05Q_{steady}$, respectively. At first, curves (τ_X) and (τ_S) show that the time to reach $Y(\tau) = 0.95Y_{eq}$ is nearly the same as that to reach $Q_S(\tau) = 1.05Q_{steady}$. The value of $(\tau_S - \tau_X)/\tau_X$ is positive, and is less than 0.004 and 0.006 in Figure 7(a)
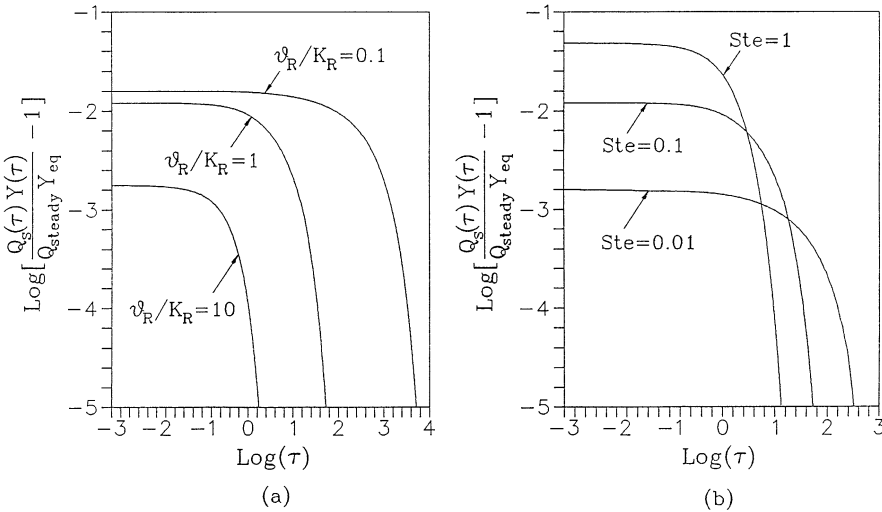
Figure 6.
Plot of $\text{Log}[Q_S(\tau)Y(\tau)/Q_{steady}Q_{steady}Y_{eq} - 1]$ with time for several values of θ_R/K_R and Ste: (a) (θ_R/K_R)-dependency, with $\alpha_R = 1$ and $Ste = 0.1$; (b) (Ste)dependency, with $\alpha_R = 1$ and $\theta_R/K_R = 1$

Figure 7.
Time required to reach $Y(\tau) = 0.95Y_{eq}(\tau_X)$, $Q_S(\tau) = 1.05Q_{steady}(\tau_S)$ and $Q_L(\tau) = 1.05Q_{steady}(\tau_L)$ as functions of θ_R/K_R and Ste: (a) (θ_R/K_R)-dependency, with $\alpha_R = 1$ and $Ste = 0.1$: (b) (Ste)-dependency, with $\alpha_R = 1$ and $\theta_R/K_R = 1$

and in (b), respectively. This result implies that the speed of approach to the equilibrium state for the solid thickness is nearly identical to that for the heat flux at the surface of solid. And comparing (τ_X) and (τ_S) with (τ_L), we can see that $Q_L(\tau)$ reaches its quasi-steady state value more rapidly than $Y(\tau)$ and $Q_S(\tau)$. When solving a solidification problem in forced fluid flow, most of the investigations assume a constant value of convective heat transfer from the liquid side of solid-liquid interface, and the transient temperature distribution in solid and the location of solid-liquid interface are determined (Savino and Siegel, 1969; Epstein, 1976). Figure 7 shows that for small θ_R/K_R or Ste, $Q_L(\tau)$ reaches its quasi-steady state value much faster than $Y(\tau)$ and $Q_S(\tau)$, and accordingly we can consider it to be constant, $Q_L(\tau) \approx Q_{steady}$, throughout the freezing process. However, the difference in the speed of approach to the equilibrium state between $Q_L(\tau)$ and $Y(\tau)$ becomes small as θ_R/K_R or Ste increases, which indicates that for large θ_R/K_R or Ste, the transient behaviour of the temperature distribution in liquid phase should be also considered.

4. Conclusions

We consider the transient heat transfer problem in the inviscid stagnation flow when phase change from liquid to solid occurs. The dimensionless governing equations have three dimensionless parameters of θ_R/K_R, Ste and α_R. The unsteady solution is found by applying the finite difference method using body-fitted coordinate, which directly determines the instantaneous temperature distribution and the solid thickness, without iterative calculation. The growth rate of solid is increased as Ste or α_R becomes large, but is decreased as θ_R/K_R increases. The equilibrium state is dependent on θ_R/K_R, but is independent of Ste and α_R. It is observed that $Q_S(\tau)/Q_{steady} \approx Y_{eq}/Y(\tau)$ throughout the freezing process where $Y(\tau)$ is the instantaneous thickness of solid phase, and $Q_S(\tau)$ is the transient heat flux at the surface of solid. For small θ_R/K_R or Ste, the transient heat flux from the liquid side of the solid-liquid interface reaches its quasi-steady state much faster than $Y(\tau)$ and $Q_S(\tau)$. However, the speed of approach to the equilibrium state for $Y(\tau)$ and $Q_S(\tau)$ becomes fast, as θ_R/K_R or Ste increases.

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